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ANGLE-OF-ARRIVAL FLUCTUATION IN BEAM
PROPAGATION THROUGH TURBULENT ATMOSPHERE

by

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# ANGLE-OF-ARRIVAL FLUCTUATION IN BEAM PROPAGATION THROUGH TURBULENT ATMOSPHERE

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ABSTRACT. In this paper, starting out near-axis approximation and considering the correlation of intensity fluctuation, one derives a universal formula (Eq. 14) of angle-of-arrival (AOA) fluctuation variance under the assumptions of a Markov random process. For plane and spherical wave propagation, analytic expressions of are obtained. Some experimental data are used to compare with analytic results. It shows that both are identical.

KEY WORDS: Turbulent flow; propagation through atmosphere, angle of arrival.

## 1. Introduction

After propagation through the atmosphere, the location of optical images on the focal plane of receiving telescopes shows random trembling due to perturbation of turbulent flow. This is the result of so-called fluctuations of beam angle of arrival. In the past two decades, a huge amount of theoretical and experimental research was focused on this subject, yet most studies were devoted to the characteristics of the weak fluctuation zone. As for the angle of arrival in the strong fluctuation zone, little was known thus far. Experiment shows that the weak perturbation theory[1] can be used to explain satisfactorily the experimental laws for weak turbulent flows. However, it will remarkably deviate from the experimental values with strong turbulent flows or long distances[2].

It is well known that under strong turbulent flows, the Markov approximation can be used to treat wave propagation equations with satisfactory results. However, while considering the correlation functions of intensity, it is possible that correct results still cannot be obtained if the second-order moment of fields is not taken into calculations. In reference[3], we tentatively discussed the angle of arrival by using Markov approximation and wave complex amplitude normal distribution approximation. In this paper, we will further discuss the expressions of angle-of-arrival variance, which are applicable to the entire fluctuation zone. In addition, we will also detail the plane wave and spherical wave propagation and compare the existing theories with experimental results. The comparison suggests that our practice here is rational.

## 2. Formula Derivation

The basic expression for the angle of arrival $\alpha_c$ is[3]

$$\vec{\alpha}_c = \frac{1}{P_0}\int_0^L d\xi \iint d^2\rho \nabla_\rho n_1 I(\xi, \vec{\rho}) T(\vec{\rho}) \tag{1}$$

where $P_0$ is total flux received, L is propagation distance, $\nabla\rho$ is the gradient operator of horizontal vector $\vec{\rho}$, $n_1$ is the fluctuation value of refractivity, $I(\xi,\vec{\rho})=u(\xi,\vec{\rho})u^*(\xi,\vec{\rho})$, $u(\xi,\vec{\rho})$ is the complex amplitude of waves, and $T(\vec{\rho})$ is the transfer function of the receiving telescope. From Eq. (1), the expression for the angle-of-arrival variance can be derived as follows: $\sigma_\alpha^2$

$$\sigma_\alpha^2 \equiv \langle \vec{\alpha}_c^2 \rangle = \frac{1}{P_0^2} \int_0^L d\xi_1 \int_0^L d\xi_2 \iint d^2\rho_1 \iint d^2\rho_2 \langle \nabla_{\rho_1} n_1 \nabla_{\rho_2} n_2 \cdot I(\xi,\vec{\rho}_1) I(\xi,\vec{\rho}_2) \rangle T(\vec{\rho}_1) T(\vec{\rho}_2) \quad (2)$$

$$P_0^2 = \iint d^2\rho_2 \iint d^2\rho_1 \langle I(\xi,\vec{\rho}_1) I(\xi,\vec{\rho}_2) \rangle T(\vec{\rho}_1) T(\vec{\rho}_2) \quad (3)$$

where <...> is the system integration average selected. Based on Markov approximation and the assumption that the fluctuation of the angle of arrival mainly comes from phase fluctuation, the following can be verified:

$$\sigma_\alpha^2 = \frac{1}{2k^2 P_0^2} \iiiint d^2\rho_1 d^2\rho_2 \nabla_\rho^2 \mathscr{D}_S(\xi,\vec{\rho}) \langle I(\xi,\vec{\rho}_1) I(\xi,\vec{\rho}_2) \rangle T(\vec{\rho}_1) T(\vec{\rho}_2) \quad (4)$$

where $k=2\pi/\lambda$; $\lambda$ is wavelength, $\mathscr{D}_S(\xi,\vec{\rho})$ is the phase structure function, which already contains the integration of distance.

To conduct a general investigation, a wave field in Gaussian form is adopted. At this instant, $u(\xi,\vec{\rho}) = u_0 \exp(-\frac{\rho^2}{2\alpha^2})$

$$\alpha^{-2} = \alpha_0^{-2} - i\frac{k}{F} \quad (5)$$

where $\alpha_0$ is the equivalent radius of laser beam at the transmitting end, and F is the curvature radius of the wavefront. Next, the transfer function $T(\vec{\rho})$ is also believed to be in Gaussian form

$$T(\vec{\rho}) = T_0 \exp(-\frac{\rho^2}{\alpha_T^2}) \quad (6)$$

where $T_0$ is penetration coefficient, $\alpha_T$ is the radius of the receiving telescope.

Under the above condition, it is also necessary to know the form of the intensity correlation function $\langle I(\xi,\vec{\rho}_1) I(\xi,\vec{\rho}_2) \rangle$ in solving Eq. (4), which used to be selected as

$$\langle I(\xi,\vec{\rho}_1) I(\xi,\vec{\rho}_2) \rangle \approx \langle I(\xi,\vec{\rho}_1) \rangle \langle I(\xi,\vec{\rho}_2) \rangle$$

This is approximately effective in the weak fluctuation zone, but in the entire fluctuation zone, the correlation of intensity fluctuation must be taken into account. We confirmed[4] that it would be appropriate to adopt the following formula:

$$\langle I(\xi,\vec{\rho}_1) I(\xi,\vec{\rho}_2) \rangle = \langle I(\xi,\vec{\rho}_1) \rangle \langle I(\xi,\vec{\rho}_2) \rangle + |\Gamma_2(\xi,\vec{\rho}_1,\vec{\rho}_2)|^2 \quad (7)$$

where $\Gamma_2(\xi,\vec{\rho}_1,\vec{\rho}_2)$ is the second-order moment (or coherent function) of the field, whose expression was given by Wang and Plonus[5] by using the quadratic approximation. Here, the coherent light source can be rewritten as follows:

$$\Gamma_2(\xi,\vec{\rho}_c,\vec{\rho}_d)=\frac{u_0^2}{a^2}\left(\frac{f}{2\alpha_0}\right)^2\exp\left(-ib\vec{\rho}_c\cdot\vec{\rho}_d-\frac{\rho_0^2}{c\alpha_0^2}-d\rho_d^2\right) \tag{8}$$

$$\vec{\rho}_c=\frac{1}{2}(\vec{\rho}_1+\vec{\rho}_2)\quad,\quad \vec{\rho}_d=\vec{\rho}_2-\vec{\rho}_1 \qquad c=\left(1-\frac{\xi}{F}\right)^2+(1+g^2)f^{-2}$$

$$a=\frac{1}{2\alpha_0^2}+\frac{1}{\rho_0^2}+\left(\frac{f}{2\alpha_0^2}\right)\left(1-\frac{\xi}{F}\right)^2 \qquad d=[3(1-\frac{\xi}{F})+(\frac{\xi}{F})^2+(1+\frac{3}{4}g^2)f^{-2}+g^{-2}]/c\rho_0^2$$

$$b=\frac{k}{\xi}\left\{1-a^{-2}\left[\left(\frac{f}{2\alpha_0^2}\right)^2\left(1-\frac{\xi}{F}\right)^2-\frac{1}{2\rho_0^2}\right]\right\} \qquad f=\frac{k\alpha_0^2}{\xi}\quad,\quad g=\frac{2\alpha_0}{\rho_0}$$

where f is the Feiner number of the transmission field, g is the specific value between beam width and the coherent length of spherical wave $\rho_0$ or the relative beam width in the turbulent atmosphere. The expression of $\rho_0$ is

$$\rho_0=(0.546C_n^2k^2L)^{-3/5} \tag{9}$$

where $C_n$ is the refractivity-fluctuation structure constant, which indicates the intensity of turbulent flows.

In $\Gamma_2(\xi,\vec{\rho}_c,\vec{\rho}_d)$ , $\vec{\rho}_d=0$ is taken as the average intensity $<I(\xi,\vec{\rho}_c)>$. By replacing $R_1=\vec{\rho}_1$ and $R_2=\vec{\rho}_2$ in Eq. (8) and introducing variables $R=\frac{1}{2}(\vec{\rho}_1+\vec{\rho}_2)$ and $\vec{\rho}=\vec{\rho}_1-\vec{\rho}_2$ , and then inserting all these together with Eq. (6) in Eq. (4), the following can be derived:

$$\sigma_x^2=\frac{u_0^4}{2k^2a^4P_0^2}\left(\frac{f}{2\alpha_0}\right)^4T_0^2\iiiint d^2\rho d^2R\nabla_\rho^2\mathscr{D}_S(\xi,\vec{\rho})$$
$$\cdot\left\{\exp\left(-\frac{2R^2+\rho^2/2}{\alpha_1^2}\right)+\exp\left(-i2bR\cdot\vec{\rho}-\frac{2R^2}{\alpha_1^2}-2d'\rho^2\right)\right\} \tag{10}$$

In the above equation, we already let

$$\alpha_1^{-2}=(c\alpha_0^2)^{-1}+\alpha_T^{-2}$$
$$d'=d+(4\alpha_T^2)^{-1}$$

By integrating variable R in Eq. (10) first and then calculating $\rho_0$, Eq. (10) will change to

$$\sigma_x^2=(2k^2)^{-1}(\alpha_1^2+\alpha_2^2)^{-1}\int_0^\infty d\rho\rho\nabla_\rho^2\mathscr{D}_S(\xi,\rho)\cdot\left\{\exp\left(-\frac{\rho^2}{2\alpha_1^2}\right)+\exp\left(-\frac{\rho^2}{2\alpha_2^2}\right)\right\} \tag{11}$$

In the calculations, we already allowed the following:

$$\alpha_2^{-2} = 4d' + b^2\alpha_1^2$$

Like $\alpha_1$, $\alpha_2$ also has a dimension of length. It can be believed that they have equivalent integrating radii of different values. Their difference is that $\alpha_2$ not only is related to beam radius and receiving radius, but also is determined by the intensity of turbulent flows.

Eq. (11) is a general expression of Gaussian beam angle-of-arrival variance. Since this kind of beam has a very complicated phase structure function, there is generally no analytic solution at all. Therefore, to understand the characteristics of $\sigma_\alpha^2$, appropriate measures have to be taken in response to specified conditions. As an example, the following discussion is devoted to plane wave and spherical wave propagation so as to ascertain the appropriateness of our measures here.

3. Plane Wave Propagation

Both theoretical analysis and practical examination suggest that the phase structure function derived based on the classical perturbation method[1] is also applicable to the strong fluctuation zone, which serves as the foundation for our discussion.

The phase structure function of plane waves can be expressed in the following equation:

$$\mathscr{D}_S(\xi, \rho) = 4\pi^2 k^2 \int_0^L d\xi \int_0^\infty dK K \Phi_n(K)[1 - J_0(\rho K)](1 + \frac{k}{K^2\xi} \sin\frac{K^2\xi}{k}) \tag{12}$$

where $\Phi_n(K)$ is the density function of turbulence spectrum, $J_0(x)$ is zero-order Bessel function. Considering $\nabla_\rho^2[1 - J_0(\rho K)] = K^2 J_0(K\rho)$, $\Phi_n(K) = 0.033 C_n^2 K^{-11/3}$ in Kolomogorov turbulence modeling. Thus, Eq. (11) changes to

$$\sigma_\alpha^2 = \frac{0.066\pi^2}{\alpha_1^2 + \alpha_2^2} \int_0^L d\xi C_n^2(\xi) \int_0^\infty d\rho \rho [\exp(-\frac{\rho^2}{2\alpha_1^2}) + \exp(-\frac{\rho^2}{2\alpha_2^2})]$$

$$\cdot \int_0^\infty dK K^{-2/3} J_0(K\rho)(1+\frac{k}{K^2\xi}\sin\frac{K^2\xi}{k}) \tag{13}$$

The first part of integration of variable K is easy to make, while its second part can also be solved through the ready integration formula after $x=K^2\xi/k$ is replaced. Based on this outcome, the integrating operation can be done for $\rho$, resulting in

$$\sigma_\alpha^2 = 2.564(2\alpha_1)^{-1/3}\int_0^L d\xi C_n^2(\xi)\frac{1+m^{5/6}}{1+m}\{1+\frac{1.16\Omega^{1/6}}{1+m^{5/6}}$$

$$\cdot \mathrm{Im}(i^{-5/6}[{}_2F_1(-\frac{5}{6},1;1;i\Omega)+m\,{}_2F_1(-\frac{5}{6},1;1;im\Omega)])\} \tag{14}$$

where we already let

$$m=\frac{\alpha_2^2}{\alpha_1^2} \quad , \quad \Omega=\frac{k\alpha_1^2}{2\xi}$$

$\mathrm{Im}(\cdot)$ suggests selecting the imaginary part of the function in the parentheses, ${}_2F_1(a_1,a_2;b;x)$ is the hypergeometric function, $\Omega$ is the Feiner number of the integration field.

Eq. (14) is the basic equation for the angle-of-arrival variance of plane waves, which can be solved as long as the relationship between turbulence intensity and distance is ascertained. To save space, here we only discuss the uniform turbulent flows. In this case, $C_n^2(\xi)$ is a constant, so Eq. (14) can be rewritten as:

$$\sigma_\alpha^2 = 2.564 C_n^2 L(2\alpha_1)^{-1/3}(\frac{1+m^{5/6}}{1+m})\left\{1+\frac{1.16\Omega^{1/6}}{1+m^{5/6}}\mathrm{Im}\left(i^{-5/6}\right.\right.$$

$$\left.\left.\cdot [{}_2F_1(-\frac{5}{6},1;1;i\Omega)+m\,{}_2F_1(-\frac{5}{6},1;1;im\Omega)]\right)\right\} \tag{15}$$

As far as the collimated laser beam with limited space is concerned, it can satisfy the condition $\rho_0 \gg \alpha_0$ when turbulent flows are extremely weak. In addition, in the far field, $(k/\xi \ll 1)\ \alpha_1 \approx \alpha_2 \approx \alpha_T$, and the specific value of the integrating radius m=1. Meanwhile, the condition of the Feiner number of the integration field $\Omega \ll 1$ can also be established. Therefore, Eq. (15) can be simplified as:

$$\sigma_0^2 = 2.56 C_n^2 L(2\alpha_T)^{-1/3} \quad (\Omega \ll 1) \tag{16}$$

This agrees closely with the results of the weak perturbation theory[1].

For the convenience of calculations, the hypergeometric function can be expressed in series. Then Eq. (15) will become a simple formula as follows: $\sigma_a^2 = \sigma_0^2 f(\Omega)$

$$f(\Omega) = \frac{1+m^{5/6}}{1+m}\{1+\frac{1.16\Omega^{1/6}}{1+m^{5/6}} \cdot [\sum_{n=0}^{\infty} a_n \Omega^{2n}(1+m^{2n+1}) + \sum_{n=0}^{\infty} b_n \Omega^{2n+1}(1+m^{2n+2})]\} \tag{17}$$

$$a_n = -a_{n-1}\frac{(2n-17/6)(2n-11/6)}{2n(2n-1)}, \quad a_0 = 1$$

$$b_n = -b_{n-1}\frac{(2n-11/6)(2n-5/6)}{2n(2n+1)},$$

$$b_0 = -0.2233$$

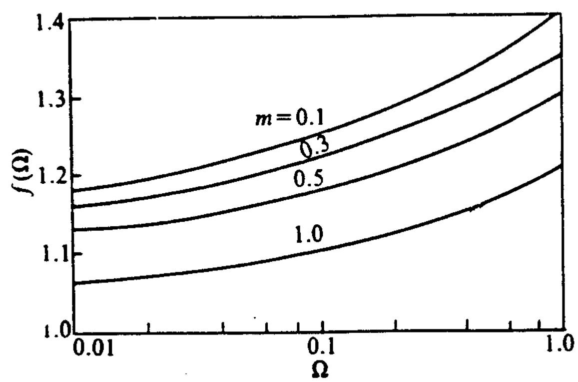


Fig. 1. f(Ω) as a function of Ω

Fig. 1 lists part of calculations conducted for function f(Ω). It is known from these calculations that in general situations, $\sigma_a^2$ is related not only to turbulence intensity $C_n^2$ , propagation distance L, and receiving aperture $\alpha_T$ as usual, but is also related to the Feiner parameters of the receiving field and the specific value of the integrating radius. This is probably because the effect of turbulent flows on light beams depends on the relative relations between beam scale and turbulence scale. While the weak perturbation theory deals with infinite plane waves, whose scale is much larger than turbulence scale, and therefore, it is not involved in such relations as mentioned here.

## 4. Spherical Wave Propagation

The phase structure function of spherical waves is

$$\mathcal{D}_S(\xi,\vec{\rho})=8\pi^2k^2\int_0^L d\xi\int_0^\infty dKK\Phi_n(K)[1-J_0(\frac{K\rho\xi}{L})]\cos^2[\frac{K^2\xi(L-\xi)}{2kL}] \tag{18}$$

Noting that $\nabla_\rho^2[1-J_0(\frac{K\xi\rho}{L})]=\frac{K^2\xi^2}{L^2}J_0(\frac{K\xi\rho}{L})$ and $\cos^2x=\frac{1}{2}(1+\cos 2x)$, under the condition of the Kolmogorov turbulence spectrum, Eq. (11) changes to

$$\sigma_\alpha^2=\frac{0.066\pi^2}{L^2}\int_0^L d\xi\,\frac{\xi^2C_n^2(\xi)}{\alpha_1^2+\alpha_2^2}\int_0^\infty d\rho\rho\{\exp(-\frac{\rho^2}{2\alpha_1^2})+\exp(-\frac{\rho^2}{2\alpha_2^2})\}$$
$$\cdot\{\frac{\Gamma(1/6)}{2^{2/3}\Gamma(5/6)}(\frac{\xi\rho}{L})^{-1/3}+\frac{\Gamma(1/6)}{2}(\frac{kL}{\xi(L-\xi)})^{1/6}\mathrm{Re}[i^{-1/6}{}_1F_1(\frac{1}{6};1;\frac{ik\xi\rho^2}{4L(L-\xi)})]\} \tag{19}$$

By further integrating variable $\rho$ and considering that $\alpha_T>>\alpha_0$ can always be satisfied in the case of spherical waves, i.e., $\alpha_1=\alpha_T$, the general expression of the angle-of-arrival variation of spherical waves is as follows:

$$\sigma_\alpha^2=2.564L^{-5/3}\int_0^L d\xi\,\frac{\xi^{5/3}C_n^2(\xi)}{(1+m_T)(2\alpha_T)^{1/3}}\{1+m_T^{5/6}+2^{-1/3}(\frac{\xi}{L-\xi}\Omega_T)^{1/6}$$
$$\cdot\mathrm{Re}(i^{-1/6}[{}_2F_1(\frac{1}{6},1;1;\frac{i\Omega_T\xi}{L-\xi})+m_T\,{}_2F_1(\frac{1}{6},1;1;\frac{im_T\Omega_T\xi}{L-\xi})])\} \tag{20}$$

where we already let

$$m_T=\frac{\alpha_2^2}{\alpha_T^2}\quad,\quad\Omega_T=\frac{k\alpha_T^2}{2L}$$

Just in the case of plane waves, Eq. (20) can also be solved as long as the function form of $C_n^2(\xi)$ is known. For the uniform optical path, we obtained

$$\sigma_\alpha^2=0.9615C_n^2L(2\alpha_T)^{-1/3}f(\Omega_T)$$
$$f(\Omega_T)=\frac{1+m^{5/6}}{1+m}\left\{1+0.09477\frac{\Omega^{1/6}}{1+m_T^{5/6}}\mathrm{Re}\left(i^{-1/6}\right.\right.$$
$$\left.\left.\cdot[G_{3,3}^{2,3}(i\Omega_T|-\frac{11}{6},\frac{5}{6},0;\frac{5}{6},0,0)+m_TG_{3,3}^{2,3}(im_T\Omega_T|-\frac{11}{6},\frac{5}{6},0;\frac{5}{6},0,0)]\right)\right\} \tag{22}$$

where $G^{m,n}_{p,q}(x|a_r;b_s)$ is the Meijer G function. The relationships between the G function and the hypergeometric function[7] were already made use of in solving Eq. (21).

The G function is extremely complex in nature. However, with it we can analyze the properties of function $f(\Omega_T)$ in the near field ($\Omega_T>1$) and far field ($\Omega_{T<1}$). Under the conditions of the near field, $m_T \approx 1+\Omega_T^2$, while

$$G^{2,3}_{3,3}(x|-\frac{11}{6},\frac{5}{6},0;\frac{5}{6},0,0)=\Gamma(\frac{8}{3})\Gamma(\frac{1}{6})x^{-1/6}{}_2F_1(1,\frac{1}{6};-\frac{5}{3};-x^{-1})$$
$$+\Gamma(-\frac{8}{3})\Gamma(\frac{11}{3})\Gamma(\frac{17}{6})x^{-17/6}{}_2F_1(\frac{11}{3},\frac{17}{6};\frac{11}{3};-x^{-1}) \qquad (x>1)$$

Then, Eq. (22) can be rewritten as

$$f(\Omega_T)=\frac{1+m_T^{5/6}}{1+m_T}\left\{1+0.79369(1+m_T^{5/6})^{-1}\mathrm{Re}\left(i^{-1/3}[{}_2F_1(1,\frac{1}{6};-\frac{5}{3};i\Omega_T^{-1})\right.\right.$$
$$+m_{T\,2}F_1(1,\frac{1}{6};-\frac{5}{3};i(m_T\Omega_T)^{-1})]-0.74698i\Omega_T^{8/3}[{}_2F_1(\frac{11}{3},\frac{17}{6};\frac{11}{3};i\Omega_T^{-1})$$
$$\left.\left.+m_{T\,2}F_1(\frac{11}{3},\frac{17}{6};\frac{11}{3};i(m_T\Omega_T)^{-1})]\right)\right\} \quad (\Omega_T>1) \tag{23}$$

Under the conditions of the far field, $m_T \approx \rho_0^2/4\Omega_T^2$, while

$$G^{2,3}_{3,3}(x|-\frac{11}{6},\frac{5}{6},0;\frac{5}{6},0,0)=\Gamma(-\frac{5}{6})\Gamma(\frac{11}{3})x^{5/6}{}_2F_1(\frac{11}{3},1;\frac{11}{6};x)$$
$$+\Gamma(\frac{5}{6})\Gamma(\frac{17}{6})\Gamma(\frac{1}{6}){}_2F_1(\frac{17}{6},\frac{1}{6};\frac{1}{6};x) \qquad (x<1)$$

Thus, Eq. (22) becomes

$$f(\Omega_T)=\frac{1+m_T^{5/6}}{1+m_T}\left\{1+1.0269\frac{\Omega_T^{1/6}}{1+m_T^{5/6}}\mathrm{Re}\left(i^{-1/6}[{}_2F_1(\frac{17}{6},\frac{1}{6};\frac{1}{6};i\Omega_T)\right.\right.$$
$$+m_{T\,2}F_1(\frac{17}{6},\frac{1}{6};\frac{1}{6};im_T\Omega_T)]-2.4733i^{2/3}\Omega^{5/6}[{}_2F_1(\frac{11}{3},1;\frac{11}{6};i\Omega_T)$$
$$\left.\left.+m_{T\,2}F_1(\frac{11}{3},1;\frac{11}{6};im_T\Omega_T)]\right)\right\} \quad (\Omega_T<1)$$

$$\tag{23'}$$

Once the Feiner number of the receiving field $\Omega_T$ and the coherent length of spherical waves $\rho_0$ are determined, function can be derived. Fig. 2 and Fig. 3 present part of calculations of $f(\Omega_T)$ . In Fig. 2, $f(\Omega_T) \to \Omega_T^{-1/3}$, under the limitation of $\Omega_T \gg 1$ , because at this point, $m_T$ is equal to $\Omega_T^2$ . In Fig. 3, only a few numbers of $m_T<1$ are selected, because in the far field, the condition of $\rho_0 < \alpha_T$ can always be satisfied.

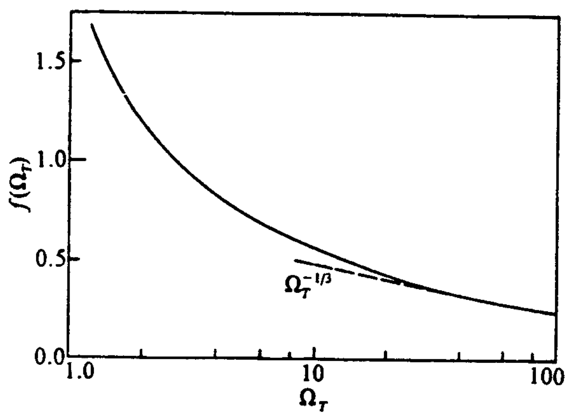


Fig. 2. $f(\Omega_T)$ as a function of $\Omega_T$ ($\Omega T>1$)

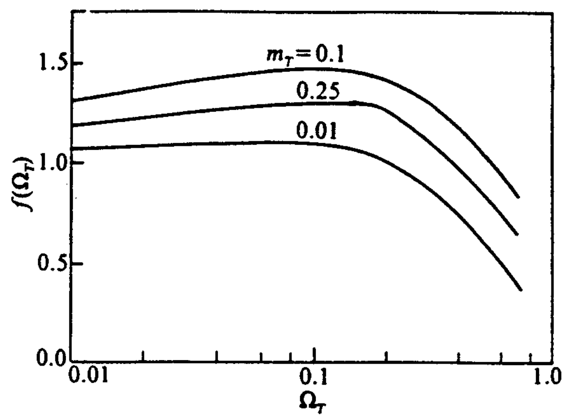


Fig. 3 $f(\Omega T)$ as a function of $\Omega_T$ ($\Omega T<1$)

To demonstrate the effectiveness of the above treatment, we made a comparison between the experimental results listed in reference[2] and the calculations. The conditions of the experiment are: $L=1750m$, $\lambda=0.5\mu m$, $\alpha_T=2.36cm$, point light source. According to the definitions given in this paper, we obtained $\Omega T=2$; we also obtained $f(\Omega_T)=0.804$ from Eq. (23). Fig. 4 shows a comparison between the experiment and the theory, where the straight line 1 is $\sigma_{sp}=1.03C_n L^{1/2}(2\alpha_T)^{-1/6}$ , derived from the weak perturbation theory, while the straight line 2 is the result of this paper. This figure indicates that the expected value in the weak perturbation theory already deviates greatly from the experimental value at $\sigma>20\mu rad$, while the results given in this paper conform to the actual situation to a greater degree. It is noteworthy that when $\sigma_{\alpha>20\mu rad}$, it is equivalent to $C_n^2=(2\sim6)\times10^{-13}m^{-2/3}$, being a mid to strong turbulence state. It can be further expected that Eq. (23) is possibly applicable to the strong fluctuation zone.

## 5. Conclusions

By using the Markov approximation, and the quadratic approximation of average intensity, we acquired the general expression of the angle-of-arrival variance and derived the analytic formula of  for plane waves and spherical waves. The practicability of these results is surely limited due to the foregoing approximation.

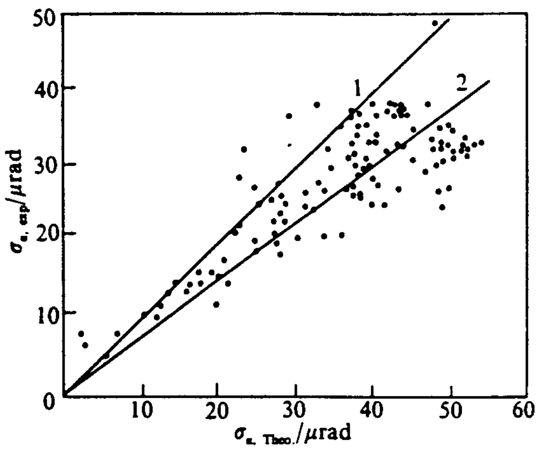


Fig. 4. Comparison of theory with experiment
KEY: 1 - small-perturbation theory 2 - our calculation

However, the Markov approximation is a traditional method dealing with wave propagation through turbulent media and has been supported by experiments[1]. As for the square approximation of average intensity, it serves as an ideal approximation according to Leader[8]. As a matter of fact, our calculation results of plane waves proved to conform to the mature theory with regard to weak turbulent flows, and our calculation results of spherical waves were supported by experiments, which suggests that the above-mentioned assumption is reasonable.

In summary, by comparing our calculations with the mature theory and the experiment, it is demonstrated that the results obtained in this paper are rational and possibly are applicable to the entire fluctuation zone. Despite this, our results are expected to be further verified due to insufficient experimental data and particularly those under different conditions.

REFERENCES

1. Tatarskiy, V. I., Rasprostranenniye voln v turbulentnoy atmosfere, Nauka Moscow.

2. Gurvich, A. S. and M. A. Kallistratova, Izv. VUZ Radiofizika, 1968, 11(1):66

3. Zhang Yexin and Song Zhengfang, Optics Journal, 1986, 6(12):1111.

4. Song Zhengfang, Basis of Applied Atmospheric Optics, Beijing, Meteorology Publishing House, 1990. 2.

5 Wang, S. C. H. and M. A. Plonus, J. Opt. Soc. Am., 1979, 69(9):1297.

6. Lukin, V. P. and V. V. Pakasov, Appl. Opt., 1981, 20(1):121.

7. Gradshteyn, I. S., and I. M. Tyzhik, Tablitsy integralov, summ, ryadov, i proizvedeniy [Tables of Integrals, Sums, Series, and Products], Nauka, Moscow, 1971:304.

8. Leader, J. C., J. Opt. Soc. Am., 1978, 68(2):175.

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